Sukanya Krishna

Clock Project Analysis

Executive Summary:

The objective of this report is to perform calculations related to point mass and rigid body analysis and compare the differences between the theoretical and experimental values. To describe the project, from the beginning, one had the freedom to choose the design of the pendulum, which was designed using the CAD software Fusion 360. Then, one learned manufacturing techniques to cut and assemble the pendulum clock. Theoretical analysis was applied to determine the accuracy of two theoretical methods used to calculate the natural frequency and time it takes for one revolution of the escapement wheel. The two methods of theoretical analysis used were the point mass method, which assumes that the mass of the pendulum is concentrated at a single point, and the rigid body method, which takes into account the pendulum's configuration and moment of inertia. According to the point mass analysis, the calculated time of one revolution of the wheel was 6.46 seconds. According to the rigid body analysis, the calculated time of one revolution was 10.13 seconds. The actual rotation of the wheel was measured to be an average of around 8.64 seconds. The percent error between the point mass analysis and the experimental value was 25.26 percent, and the percent error between the rigid body analysis and the experiment value was 17.22 percent, which indicated that the rigid body analysis was more accurate than the point mass analysis.

Theoretical Analysis:

The components of the clock include the escapement wheel and the pendulum. The escapement wheel is attached to a system with a counterweight (two nuts). When the counterweight is allowed to fall to the ground, the escapement wheel will convert conserved potential energy into rotational energy, which provides the wheel with enough torque to spin. The wheel will continue rotating because its teeth are made to interact with the two pallets of the pendulum: the locking face and the impulse face (Figure 1). The locking face (right pallet) locks the tooth of the clock in place and does not apply any force to the pendulum. The impulse face (left pallet) is designed to receive a push from the escapement wheel [1]. Both pallets are designed to keep the wheel moving.

The escapement wheel is attached to a counterweight which falls to the ground and converts that conserved potential energy into rotational energy to provide the wheel with enough torque to spin.

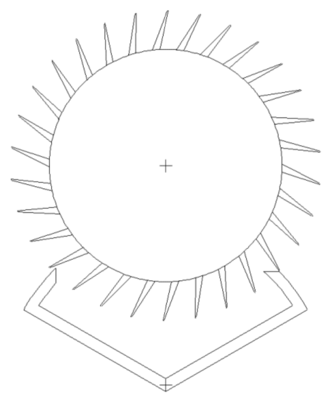

Figure 1: Escapement Wheel Layout with Pallets [1]

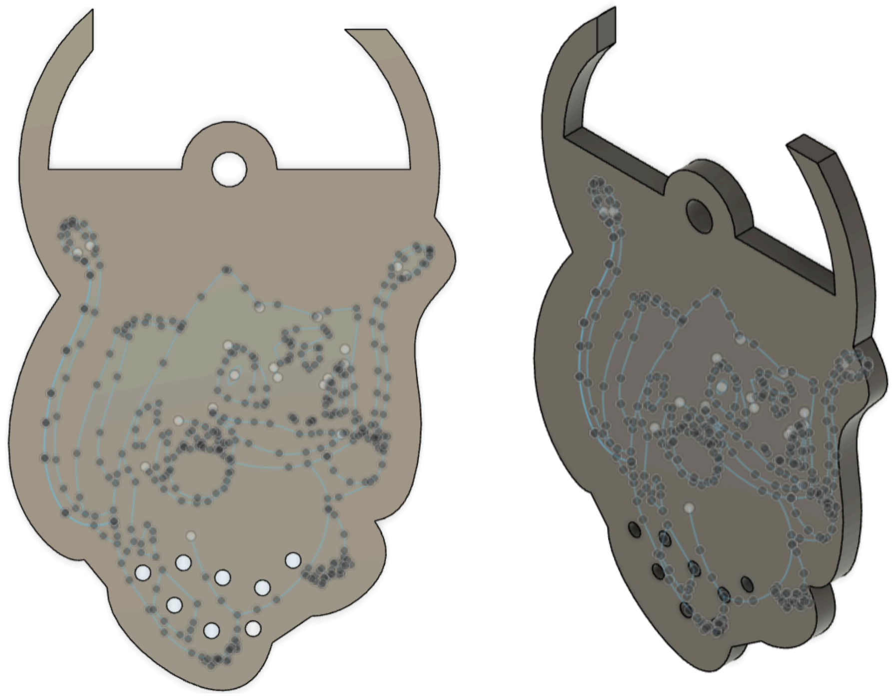

Figure 2: CAD Pendulum Front View and Right Isometric View from AutoCAD Fusion 360

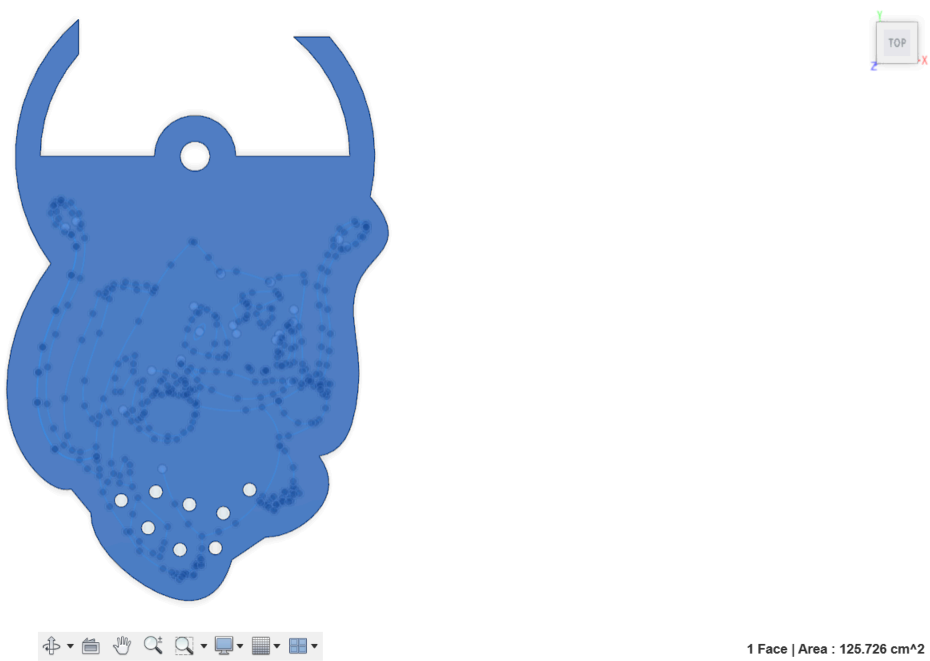

Figure 3: Surface Area from CAD region analysis of Pendulum

The clock timing analysis of the pendulum (Figure 2) was analyzed using two theoretical methods: the point mass analysis, which approximates the mass of the pendulum as occurring at a single point, and the rigid body analysis which treats the pendulum as a rigid body with a moment of inertia along with its mass. In addition to assuming that the mass of the pendulum is concentrated at a single point, the point mass method also assumes that there is no friction, there is a small angle of oscillation, the pendulum rope doesn't stretch, and that the pendulum frequency is not influenced by the escapement wheel [1].

For predicting the acrylic pendulum's specifications that were utilized for both theoretical analyses, the area and the mass of the acrylic for the point mass approach were calculated using Fusion 360 (Figure 3). The area of the acrylic was determined to be 125.726 cm^2. With the known thickness of 0.635 cm (0.25 in), the volume of the acrylic was found using Equation 1 where A represented the area and t represented the thickness of the pendulum design. As such, the Volume (V) is calculated to be 79.836 cm^3.

$$V = A * t \qquad (1)$$

To calculate the predicted mass of the pendulum, Equation 2 was used where V is the volume and ρ is the density of the acrylic, which is 1.188 $\frac{g}{cm^3}$. The predicted mass of the pendulum was 94.85 grams. The predicted mass of the pendulum with nuts and bolts (M_t) using Equation 3 was 126.85 grams where N_b, the number of bolts, was set to 8, and M_b, the mass of a set of nuts and bolts, was set to 4 grams [2].

$$M_{Calc} = V * \rho \qquad (2)$$

$$M_t = M_{Calc} + N_b * M_b \qquad (3)$$

The predicted effective length of the center of mass of the pendulum was estimated using two different methods. The first method was based on real-life measurement L_a, which involved approximately measuring the length from the centroid of the pendulum to the center of mass directly using a finger. An assumption was that this value was appropriate because a finger was used to balance the pendulum and approximate the point where the pendulum was most stable. The length from the centroid to that point was measured using a ruler and L_a was estimated to be around 2.54 cm. This was an intermediate verification step, as this value was utilized to calculate the estimated length to the center of mass using Equation 4 [2].

The equation for the first method is detailed in Equation 4. This method involved measuring the distances of the pendulum pivot point to each bolt (Figure 4), and utilizing the total mass of the pendulum, M_t, calculated using Equation 3. In Equation 4, M_{Calc} is the value determined from applying Equation 2, L_a is the measured length from the pendulum pivot to the center of the mass of the pendulum, $L_{bolt,\,i}$ is the measured length from the pivot point to each bolt, N_b is 8, representing the number of bolts, and M_b is 4, representing the mass of each bolt in grams. $L_{com,\,estimated}$ was calculated to be 0.062 meters [2].

$$L_{com,\,estimated} = \frac{M_{Calc}(L_A) + M_b(\Sigma L_{bolt,\,i})}{M_t} \qquad (4)$$

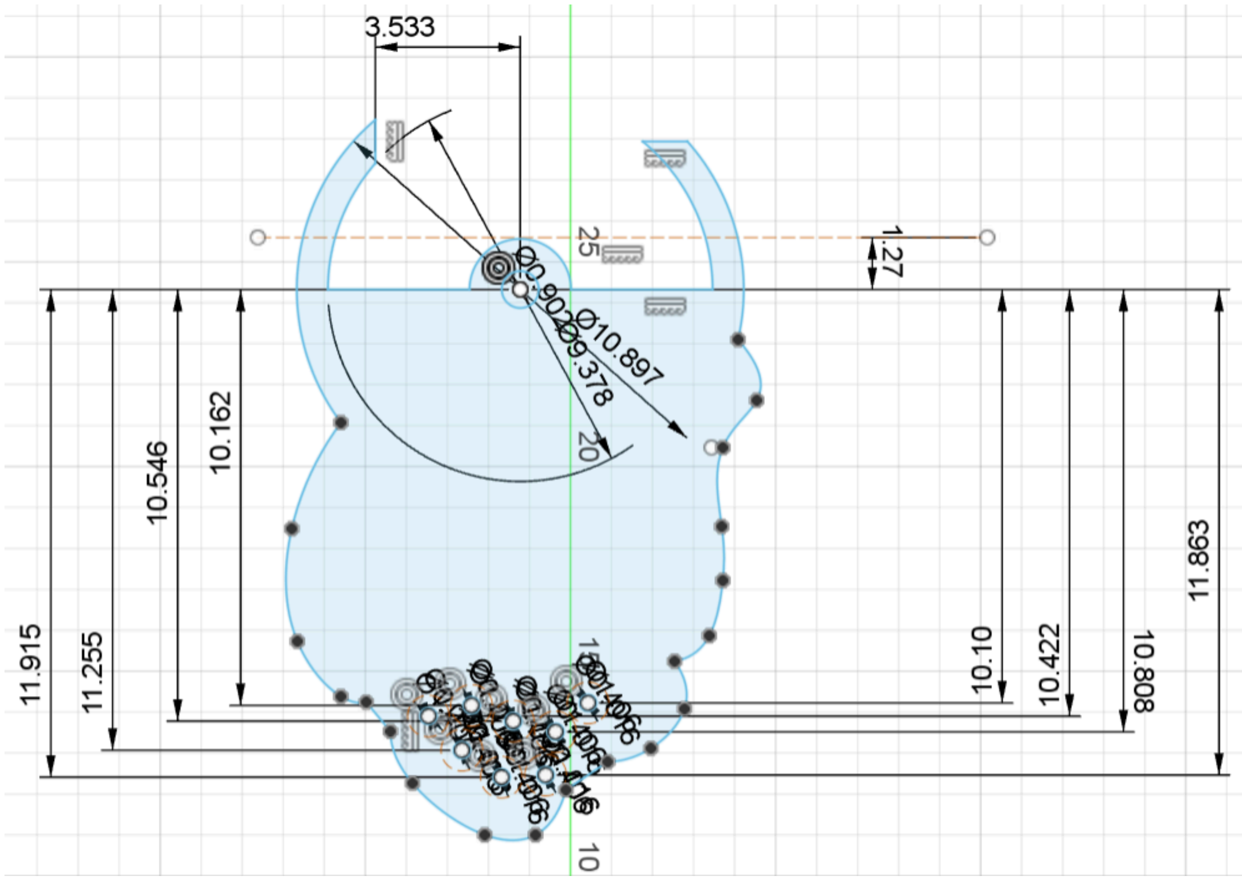

Figure 4: Using the Dimension Tool in Fusion 360 to determine the vertical length of each bolt from the centroid of the pendulum

The second method of finding the length from the pivot point to the center of mass was getting this measurement directly from Fusion 360 (Figure 5). Using the properties tool, the length to the center of mass, $L_{com,\, meter}$, was estimated to be the vertical distance or the y-component and was determined to be 0.053 meters [2].

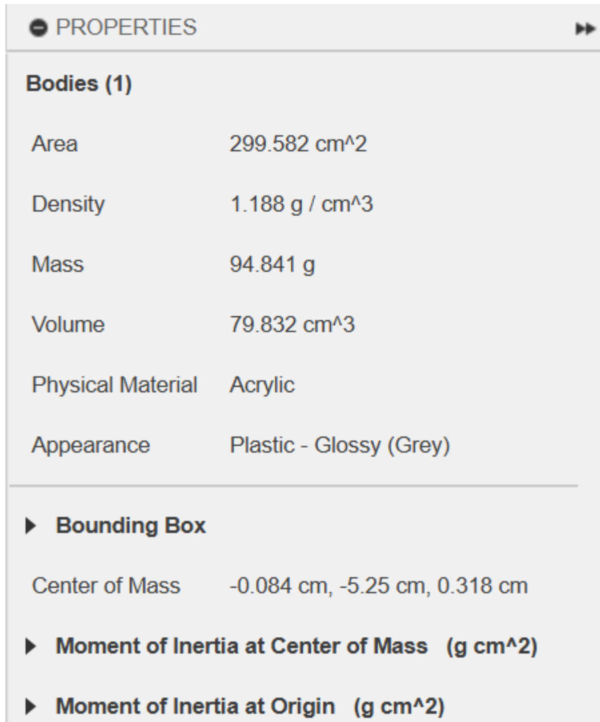

Figure 5: Using the properties tool in Fusion 360 to determine the length to the center of mass

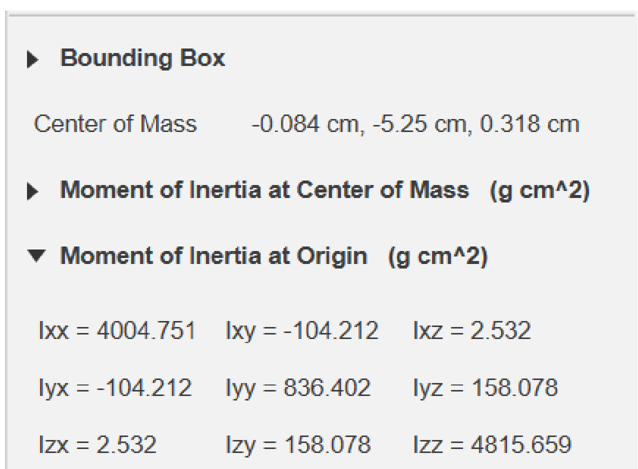

Figure 6: Moment of Inertia about the Origin

The assumptions for the rigid body analysis are the same as those for the point mass analysis, however, the rigid body analysis does not assume that the mass of the pendulum concentrates at a point. Rather, the rigid body analysis accounts for the uneven distribution of mass along the body [1]. The inertia of the pendulum (I_a) was determined using Fusion 360 (Figure 6), only observing the Izz component, which is along the axis normal to the plane that the pendulum was constructed on (xy plane). As such, the pendulum would be rotating about the z-axis, and the inertia was 4847.446 $gm * cm^2$ [3].

$$I_{total} = I_a + \Sigma I_{bolt, i} \qquad (5)$$

Equation 5 was used to calculate the overall inertia of the pendulum where I_a is the inertia of the acrylic given by Fusion 360 and I_{Bolt} is the inertia of the bolts attached to the pendulum. $I_{bolt, i}$, was calculated using the length between the bolt and the pendulum pivot point as the radius, and the mass of the bolt, which was 4 grams. These values were plugged into the formula for inertia in Equation 6, to calculate each $I_{bolt, i}$ [3]. I_{total} was found to be 0.870 $g * m^2$.

$$I = m * r^2 \qquad (6)$$

All of the theoretical measurements determined above were utilized for predicting the natural frequency of the pendulum and calculated time for one revolution of the escapement wheel according to both analysis methods. For the point mass analysis, the natural frequency was predicted to be 13.62 rad/s following Equation 7, where g is gravity [2]. For the rigid body analysis, the natural frequency was predicted to be 8.69 rad/s following Equation 8 [3].

$$\omega = \sqrt{\frac{g}{L_{com, meter}}} \qquad (7)$$

$$\omega = \sqrt{\frac{M_t * g * L_{com, meter}}{I_{total}}} \qquad (8)$$

For both analysis methods, the natural frequency of the pendulum in Hertz is calculated using Equation 9, the period of oscillation using Equation 10, and the predicted time of one revolution of the escapement wheel is calculated in Equation 11 using the period from Equation 9 and the number of teeth on the escapement wheel [2]. This is because based on the configuration of the pendulum clock, the clock will make one swing per tooth on the escapement wheel. As there are 14 teeth on the escapement wheel (n_{teeth}), the time per revolution boils down to the number of teeth for one full rotation of the escapement wheel.

$$f = \frac{\omega}{2\pi} \qquad (9)$$

$$T = \frac{1}{f} \qquad (10)$$

$$t = T * n_{teeth} \qquad (11)$$

For the point mass analysis, the theoretical frequency in Hz is 2.168 Hz, one period of oscillation is 0.461 s/cycle, and the time for one revolution of the escapement wheel is 6.457 s/revolution. For the rigid body analysis, the theoretical frequency in Hz is 1.382 Hz, one period of oscillation is 0.723 s/cycle and the time for one revolution of the escapement wheel is 10.128 s/revolution.

In the Appendix, a view of the Excel utilizing all the numerical results and calculations can be found.

Experimental Results:

Determining time_meas	
Trial Number	time (s)
1	9.5
2	8.3
3	8.7
4	9.2
5	8.2
6	8.3
7	9.1
8	8.6
9	8.6
10	7.9
Average	8.64
Standard Deviation	0.4993328883

Table 1: Trials for the amount of time it takes the pendulum clock to complete one revolution

The actual mass of the pendulum was measured using a scale and was found to be around 76 grams. Against the calculated mass of the acrylic, 94.8 grams from applying Equation 2, the percent error between the actual and predicted mass was 24.8 percent.

To determine the actual amount of time it would take for the pendulum clock to complete one revolution, 10 trials were conducted (Table 1). The time for a full rotation was recorded using a stopwatch after the counterweight was released. The average of the trials was calculated to be 8.64 seconds and the standard deviation was calculated to be 0.50 seconds.

The percent error as a percentage for the experimental time to the actual time was determined using Equation 12.

$$t = \frac{(t_{measured} - t_{calculated})}{t_{measured}} * 100 \qquad (12)$$

The percent error between the point mass analysis and the experimental value was 25.26 percent, and the percent error between the rigid body analysis and the experiment value was 17.22 percent, which for this clock design indicated that the point mass analysis was more accurate than the rigid body analysis.

Discussion:

The point mass and the rigid body analysis methods were utilized to analyze the natural frequency and time it would take for one revolution of the clock escapement wheel. According to the experimental trials run on the finished clock pendulum, it took approximately 8.64 seconds for one revolution of the escapement wheel. Both analysis methods led to different calculated times in that according to the point mass analysis, it took 6.46 seconds per revolution, which leads to a percent error of 25.26 percent with the experimental result (Table 2). According to the rigid body analysis, it took 10.13 seconds for one rotation of the wheel, which leads to a percent error of 17.22 percent with the experimental result (Table 3). Similarly, both analysis methods showed a discrepancy in the calculated natural frequency. According to the point mass analysis, the natural frequency is 13.62 rad/s, and according to the rigid body analysis, the natural frequency is 8.69 rad/s [2]. As such, it is indicated that the frequency of the pendulum is faster for the point mass analysis than the rigid body analysis, which is reflected in the final calculated times for one revolution of the escapement wheel [2].

Both analysis methods utilized the length to the center of mass for their natural frequency calculations. There were two methods used to calculate the center of mass for the pendulum. The first method, which estimated the length to the center of mass (L_{com_est}) based on the real-life measurement, L_a and the center of mass of the bolts, was calculated to be 0.062 meters. The second method was the length obtained from fusion, which can be related directly to the calculated time of one revolution, and was calculated to be 0.053 meters. The percent error was calculated and found to be 17.63 percent.

Observing the percent errors for the time it takes for one revolution of the escapement wheel, it can be inferred that the rigid body analysis method is more accurate than the point mass analysis method. This could be attributed to the fact that the rigid body analysis takes into account all of the individual moments that the bolts bring rather than just assuming that the mass of the pendulum can be concentrated into a point mass as the point mass analysis does. As such, the point mass analysis does not well represent the overall conformation and distribution of the pendulum's mass since each bolt is at a different distance from the clock pivot point, and so would bring a unique inertia that would be added to the overall inertia of the pendulum. Because the rigid body analysis accounts for the pendulum's need to overcome its inertia, the natural frequency and timing will be slower compared to those in the point mass analysis.

Several factors contribute to the disparities between the calculated and experimental values. Firstly, one main assumption was the omission of air resistance and friction between the pendulum's components. As these forces would hinder the pendulum's movement, this would result in a greater experimental value for one rotation compared to the calculated value. The neglect of the escapement wheel's inertia is another factor. The inertia of the escapement wheel wasn't accounted for, however, the escapement wheel itself also contains some mass and thereby generates an inertia that needs to be overcome for rotation. As such, this oversight causes the wheel to rotate at a slower pace, leading to higher experimental values compared to the theoretical ones.

Two additional reasons that may contribute to the discrepancy in the calculated and theoretical rotation times of the clock are that during the fabrication process, there may have been imperfections in placing the acrylic sheet on the bed, causing an uneven surface. This, in turn, can lead to imprecise cuts at the pivot point, resulting in an angular position when the pendulum component is assembled on the clock's small bar. The other reason is a lack of precision during the fabrication process where the small bar was not perfectly inserted into the upright, causing a slight slant. Since the pendulum is not perfectly perpendicular to the surface of the upright, this misalignment introduces some errors as well.

Appendix:

Excel tables with numerical results and calculations

Pendulum Timing Analysis				
Name: Sukanya Krishna				
Section: A06				
Variable Description	**Variable Name**	**Values/Equations**	**Units**	
Acrylic Pendulum Specifications				
Area	A	125.726	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	79.83601	cm^3	
Density	p	1.188	gm/cm^3	
Calculated Mass of Acrylic	M_Calc	94.84517988	gm	
Actual Mass of Acrylic	M_Act	76	gm	
Length to Center of Mass of Acrylic	La	2.54	cm	
Percent Error in Acrylic Mass Calculation	M_Error	24.79628932	%	
Calculate Total Mass of Pendulum				
Mass of One Bolt with Two Nuts	Mb	4	g	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	126.8451799	g	
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	10.1	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	10.422	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	10.808	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	11.863	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	10.162	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	10.546	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	11.255	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	11.915	cm	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	0.05281	**meters**	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est	0.06212131788	meters	
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error	17.6317324	%	

Calculate Natural Frequency and Timing using Point Mass Assumption			
Gravitational Constant	g	9.8	m/s^2
Natural Frequency in radians/sec	nat_freq_rad_sec	13.62244158	rad/s
Natural Frequency in Hz	nat_freq_hz	2.168078914	Hz
Period of Oscillation	period	0.4612378237	s/cycle
Number of Teeth on Escapement Wheel	nteeth	14	swings
Calculated Time of One Revolution of Escapement Wheel	time_calc	6.457329531	s/rev
Measured Time of One Revolution of Escapement Wheel	time_meas	8.64	s/rev
Percent Error in Clock Timing	time_error	25.26238969	%
Calculate Natural Frequency and Timing using Rigid Body Assumption			
Moment of Inertia of Pendulum	I_a	4847.446	g*cm^2
Moment of Inertia of Bolt 1	I_bolt1	419.0209	g*cm^2
Moment of Inertia of Bolt 2	I_bolt2	444.872464	g*cm^2
Moment of Inertia of Bolt 3	I_bolt3	470.195856	g*cm^2
Moment of Inertia of Bolt 4	I_bolt4	564.442564	g*cm^2
Moment of Inertia of Bolt 5	I_bolt5	454.5424	g*cm^2
Moment of Inertia of Bolt 6	I_bolt6	418.693444	g*cm^2
Moment of Inertia of Bolt 7	I_bolt7	514.8361	g*cm^2
Moment of Inertia of Bolt 8	I_bolt8	568.727104	g*cm^2
Total Moment of Inertia	I_total	0.8702776832	g*m^2
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	8.685187177	rad/s
Natural Frequency in Hz	rb_nat_freq_hz	1.382290471	Hz
Period of Oscillation	rb_period	0.7234369484	s/cycle
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	10.12811728	s/rev
Percent Error in Clock Timing	rb_time_error	17.2235796	%

Table 2: All values calculated for point mass and rigid body assumption

Pendulum Timing Analysis

Name: Sukanya Krishna
Section: A06

Variable Description	Variable Name	Values/Equations	Units	
Acrylic Pendulum Specifications				
Area	A	125.726	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	=A*t	cm^3	=C8*C9
Density	p	1.188	gm/cm^3	
Calculated Mass of Acrylic	M_Calc	=Vol*p	gm	
Actual Mass of Acrylic	M_Act	76	gm	
Length to Center of Mass of Acrylic	La	=1*2.54	cm	
Percent Error in Acrylic Mass Calculation	M_Error	=abs((M_Act-M_Calc)/M_Act * 100)	%	
Calculate Total Mass of Pendulum				
Mass of One Bolt with Two Nuts	Mb	4	g	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	=M_Calc + Nb * Mb	g	=76+32
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	10.1	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	10.422	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	10.808	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	11.863	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	10.162	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	10.546	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	11.255	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	11.915	cm	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	=5.281/100	**meters**	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est	=(((M_Calc*La) + Mb*(sum(L_bolt1:L_bolt8)))/M_Calc)/100	meters	
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error	=abs((Lcom_meter-Lcom_est)/Lcom_meter * 100)	%	

Calculate Natural Frequency and Timing using Point Mass Assumption			
Gravitational Constant	g	9.8	m/s^2
Natural Frequency in radians/sec	nat_freq_rad_sec	=sqrt(g/Lcom_meter)	rad/s
Natural Frequency in Hz	nat_freq_hz	=nat_freq_rad_sec/(2*pi())	Hz
Period of Oscillation	period	=1/nat_freq_hz	s/cycle
Number of Teeth on Escapement Wheel	nteeth	14	swings
Calculated Time of One Revolution of Escapement Wheel	time_calc	=period*nteeth*1	s/rev
Measured Time of One Revolution of Escapement Wheel	time_meas	8.64	s/rev
Percent Error in Clock Timing	time_error	=abs((time_meas - time_calc)/time_meas * 100)	%
Calculate Natural Frequency and Timing using Rigid Body Assumption			
Moment of Inertia of Pendulum	I_a	4847.446	g*cm^2
Moment of Inertia of Bolt 1	I_bolt1	=4*(10.235)^2	g*cm^2
Moment of Inertia of Bolt 2	I_bolt2	=4*(10.546)^2	g*cm^2
Moment of Inertia of Bolt 3	I_bolt3	=4*(10.842)^2	g*cm^2
Moment of Inertia of Bolt 4	I_bolt4	=4*(11.879)^2	g*cm^2
Moment of Inertia of Bolt 5	I_bolt5	=4*(10.66)^2	g*cm^2
Moment of Inertia of Bolt 6	I_bolt6	=4*(10.231)^2	g*cm^2
Moment of Inertia of Bolt 7	I_bolt7	=4*(11.345)^2	g*cm^2
Moment of Inertia of Bolt 8	I_bolt8	=4*(11.924)^2	g*cm^2
Total Moment of Inertia	I_total	=sum(I_a:I_bolt8)/10000	g*m^2
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	=sqrt((Mt*g*Lcom_meter)/I_total)	rad/s
Natural Frequency in Hz	rb_nat_freq_hz	=rb_nat_freq_rad_sec/(2*pi())	Hz
Period of Oscillation	rb_period	=1/rb_nat_freq_hz	s/cycle
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	=rb_period*nteeth*1	s/rev
Percent Error in Clock Timing	rb_time_error	=abs((time_meas - rb_time_calc)/time_meas * 100)	%

Table 3: All calculations done for point mass and rigid body assumption

References:

[1] Morimoto, Tania. “Lecture 2 - Clock Design.” MAE 3, University of California San Diego, October 3, 2023.

[2] Morimoto, Tania. “Tutorial 6 (Clock): Natural Frequency Using Point Mass” MAE 3, University of California San Diego, October 11, 2023.

[3] Morimoto, Tania. “Tutorial 7 (Clock): Natural Frequency Using Rigid Body” MAE 3, University of California San Diego, October 11, 2023.